What you will learn about:
Inverse Functions

Vertical Line test for Function

Horizontal Line test for Inverse

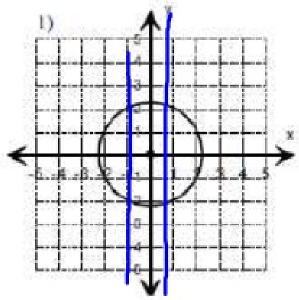
Inverse switch X and Y

Graph is reflection over line Y=X

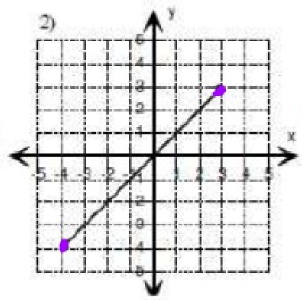
Is the relation a function? Does the relation have an inverse? If the function has an inverse, sketch the graph of the inverse.

(3,3)

(-4,-4)

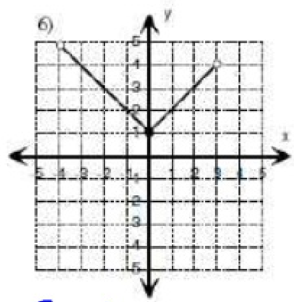


No Function



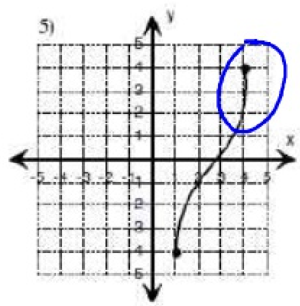
Function

Inverse



Function

No Inverse



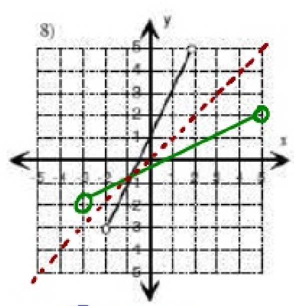
No Function

(2,5)

(5,2)

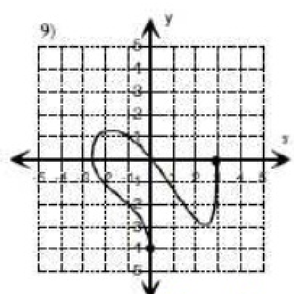
(-2,-3)

(-3,-2



Function

Inverse



No Function

Inverse

f^{-1}

Find inverse
switch x + y
solve for y

$f^{-1}(x) = \frac{x-2}{5}$

$x+5 \geq 0$

$x \geq -5$

$2 \cdot 2 \cdot 2 = 8$

$2^3 = 8$

$\sqrt[3]{8} = 2$

Find a formula for $f^{-1}(x)$. Give the domain of $f^{-1}(x)$, including any restrictions "inherited" from f.

D: $(-\infty, \infty)$

A. f(x) = 5x + 2

$y = 5x + 2$

$x = 5y + 2$

$\frac{x-2}{5} = \frac{5y}{5}$

$y = \frac{x-2}{5}$

$f^{-1}(x) = \frac{1}{5}x - \frac{2}{5}$

D: $(-\infty, \infty)$

D: $(-\infty, 1) \cup (1, \infty)$

B. $f(x) = \frac{3x+2}{x-1}$

$(y-1)(x) = \left(\frac{3y+2}{y-1}\right) y-1$

$xy - x = 3y + 2$

$xy - 3y = x + 2$

$y(x-3) = x + 2$

$f^{-1}(x) = \frac{x+2}{x-3}$

D: $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$

C. $f(x) = \sqrt{x+5}$

D: $[-5, \infty)$

$(x)^2 = \left(\sqrt{y+5}\right)^2$

$x^2 = y + 5$

$f^{-1}(x) = x^2 - 5$

D: $[-5, \infty)$

D. $f(x) = \sqrt{x^3 - 2}$

D: $(-\infty, \infty)$

E. $f(x) = \sqrt[3]{2x+1}$

$(x)^3 = \left(\sqrt[3]{2y+1}\right)^3$

$x^3 = 2y + 1$
${-1} \qquad {-1}$

$2y = x^3 - 1$

$f^{-1}(x) = \frac{x^3 - 1}{2}$

D: $(-\infty, \infty)$

$f(g(x)) \rightarrow$ f of g of x

| Composite

$f(x) = x^3 + 1$ | Confirm that f and g are inverses by showing that f(g(x)) and g(f(x)) = x.

A. $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$

$f(g(x)) = x^3 + 1$
$f(\sqrt[3]{x-1}) = (\sqrt[3]{x-1})^3 + 1$
$= x - 1 + 1$
$= x$

$g(f(x)) = \sqrt[3]{x-1}$
$g(x^3+1) = \sqrt[3]{x^3+1-1}$
$= \sqrt[3]{x^3}$
$= x$

f(x) and g(x) are inverses

$f(x) = \frac{x+3}{x-2}$ and $g(x) = \frac{2x+3}{x-1}$

$f(g(x)) = \frac{x+3}{x-2}$
$f\left(\frac{2x+3}{x-1}\right) = \frac{\frac{2x+3}{x-1} + \frac{3}{1}}{\frac{2x+3}{x-1} - \frac{2}{1}} = \frac{\frac{2x+3}{x-1} + \frac{3(x-1)}{x-1}}{\frac{2x+3}{x-1} - \frac{2(x-1)}{(x-1)}}$

$\frac{\frac{2x+3}{x-1} + \frac{3x-3}{x-1}}{\frac{2x+3}{x-1} - \frac{2x-2}{x-1}}$ |

$\frac{\frac{5x}{x-1}}{\frac{5}{x-1}} \cdot \frac{x-1}{5}$

x